NUMERICAL STUDY OF MHD NATURAL CONVECTION IN AN INCLINED RECTANGULAR CAVITY WITH INTERNAL HEAT GENERATION FILLED WITH A POROUS MEDIUM UNDER THE INFLUENCE OF JOULE HEATING

S. E. AHMED

Abstract — Using an implicit finite difference method, the effect of Joule heating on the magnetohydrodynamic natural convection in an inclined rectangular enclosure with internal heat generation has been performed. The left and right walls of the enclosure are maintained at a constant temperature while the bottom and top walls are kept insulated. The values of the governing parameters are the Hartmann number, \( H_a = 0, 5, 10 \) and 50, Joule heating parameter \( J = 0, 0.005, 0.01 \) and 0.02, the aspect ratio \( a = 0.01, 0.2, 0.5 \) and 1, the inclination angle \( \phi = 0, \pi/6 \) and \( \pi/3 \) and Rayleigh number \( Ra = 10^2 \) and \( 10^3 \). Comparisons with previously published work are performed and excellent agreement is obtained.

A parametric study of the physical parameters is conducted and a representative set of numerical results for the streamlines and isotherms as well as the local Nusselt number is illustrated graphically to show interesting features of the solutions. It is found that the local Nusselt number increases on the bottom wall as the enclosure aspect ratio increases.

Keywords — Internal and Joule Heating, MHD natural convection, Heat generation, Numerical simulation, Inclined cavity, Porous medium.

I. INTRODUCTION

The study of natural convection in fluid saturated porous media has several applications in engineering and nature. Such applications include exothermic reactions in packed bed reactors, heat transfer associated with the deep storage of nuclear waste, flow past heat exchanger tubes and to study the effect of the metabolic heat generation in grains causing hot spots that induce fungal growth (Nield and Bejan, 2006; Ingham and Pop, 2005; Vafai, 2005; Bejan et al., 2004; Pop and Ingham 2001; Prasad, 1987; Jimenez et al., 1999; Nithiarasu et al., 2000). There are many important effects should not be neglected due to investigating heat transfer inside the enclosures filled with porous media. Such effects include magnetic force effect, Joule heating effect and viscous dissipation effect. Recently, the equally important problem of hydromagnetic convective flow of a conducting fluid through a porous medium has been investigated. In fact, when an electrically conducting fluid is subjected to a magnetic field, the fluid motion induces an electric current and, in general, the fluid velocity is reduced due to the induced Lorentz force. There are many open literature related to natural convection with magnetic force. Garandet et al. (1992) investigated buoyancy-driven convection in a rectangular enclosure with a transverse magnetic field. Hydromagnetic free convection flow through a porous medium between two parallel plates was investigated by Raptis et al. (1982a). Raptis and Vlahos (1982) further extended their investigation to study the free convection flow of a conducting fluid through a porous medium bounded by two horizontal plates. Singh and Diukshit (1987) studied the free convection of the Couette motion of an electrically conducting fluid through a porous medium. Exact solutions for the velocity field, skin-friction, and the volume flux of the fluid were obtained in terms of the governing parameters of the problem. Haajizadeh et al. (1984) investigated natural convection in a vertical porous enclosure with internal heat generation. Grosan et al. (2009) extended the Haajizadeh et al. (1984) problem by considering the effect of an inclined magnetic field.

In this study, the effect of Joule heating on MHD natural convection in an inclined rectangular cavity filled with a porous medium is considered. These types of the problems are well known natural phenomenon and have attracted interest of many researchers due to its many practical situations. Among these insulation materials, the electronic packages and microelectronic devices. The problem of steady conjugate heat transfer through an electrically-conducting fluid for a vertical flat plate in the presence of transverse uniform magnetic field taking into account the effects of viscous dissipation, Joule heating, and heat generation was formulated by Azim et al. (2010). Zhao and Yang (2011) presented an analysis of Joule heating induced heat transfer for electroosmotic flow (EOF) of power-law fluids in a microcapillary. A comprehensive investigation on hydrodynamic and thermal transport properties of mixed electroosmotically and pressure driven flow in microtubes was presented by Yavari et al. (2012). Rahman et al. (2010) studied the conjugate effect of Joule heating and magnetic force, acting normal to the left vertical wall of an obstructed lid-driven cavity saturated with an electrically conducting fluid.

II. MATHEMATICAL FORMULATION

Consider steady laminar natural convection in an inclined rectangular cavity with an electrically conducting fluid saturated porous medium. It is assumed that, the height of the cavity is denoted by \( h \), the width is denoted by \( l \) and the inclination angle by \( \phi \). The left and right walls are maintained at a constant temperature \( T_0 \), while the top and bottom walls are insulated. The enclosure is permeated by a uniform magnetic field. A uniform source of heat generation with constant volumetric rate...