FULLY DEVELOPED LAMINAR CONVECTION WITH VARIABLE THERMOPHYSICAL PROPERTIES BETWEEN TWO HEATED VERTICAL PARALLEL PLATES

D. F. DELMASTRO†, A. F. CHASSEUR‡ and J. C. GARCÍÁ§

†Centro Atómico Bariloche and Instituto Balseiro, 8400 Bariloche, Argentina. delmasto@cab.cnea.gov.ar
‡ Centro Atómico Bariloche, 8400 Bariloche, Argentina. alfredo.chasseur@cab.cnea.gov.ar
§ Centro Atómico Bariloche and Instituto Balseiro, 8400 Bariloche, Argentina. garciajc@cab.cnea.gov.ar

Abstract — In this work, the influence on the flow and heat transfer of the density, viscosity and thermal conductivity temperature dependence was analyzed. A perturbation analysis was applied to a vertical fully developed laminar stationary flow between two heated parallel flat plates. For small values of the dimensionless numbers associated with the buoyancy, viscosity and thermal conductivity changes effects, the influence of the different properties temperature dependence was obtained. Application examples to water and air flows are also presented.

Keywords — Laminar flows, heat transfer, temperature-dependent properties.

I. INTRODUCTION

In many situations of heat transfer applications, like research reactors fuel elements, a coolant fluid flows between heated vertical parallel flat plates. In case of low flow rates and significant heat fluxes, laminar flows with important temperature and properties variations appear. For these temperature-dependent property problems it is useful to know the influence of these variations on the flow and heat transfer behaviors.

One complication of the temperature-dependent property problem is that the properties of different fluids change differently with temperature. For most liquids the viscosity decrease a lot with temperature, the density varies little, while the specific heat and thermal conductivity are relatively independent of temperature. For gases the viscosity and the thermal conductivity increase very much with temperature while density decrease and specific heat varies only slightly. For engineering applications two schemes for correction of the constant-property results are usually used. In the reference temperature method a characteristic temperature is used to evaluate the properties. In the property ratio method all the properties are evaluated at the mean temperature.

The internal laminar heat transfer with temperature dependent properties were numerically investigated using finite difference methods (Swearingen and McEligot, 1971).

Many solutions have been obtained for laminar flow taking into account the effects of temperature-dependent viscosity or density. But generally they do not consider both effects nor the thermal conductivity temperature dependence (Bergles, 1983).

Recently the fully developed laminar free convection with large temperature differences was numerically studied for three fluids taking into account the variation of density, viscosity and thermal conductivity with temperature (Pantokratoras, 2006).

In this work, the influence on the flow and heat transfer of the density, viscosity and thermal conductivity temperature dependence for a mixed convection case is analyzed. A perturbation analysis is applied to a vertical fully developed laminar stationary flow between two heated parallel flat plates. For small values of the dimensionless numbers associated with the buoyancy, viscosity and thermal conductivity changes effects, the linear influence of the temperature-dependent properties is obtained.

II. PERTURBATIVE METHOD

A. Laminar Flow between Two Parallel Heated Flat Plates

Let us consider a fully developed laminar stationary flow between two parallel flat plates a distance \( 2L \) apart (Fig. 1). Both plates have a uniform heat flux \( q' \). The flow is vertical, upward or downward. The \( x \) coordinate is vertical and positive downward. The \( y \) coordinate is horizontal and perpendicular to the plates.

Let us assume that the density \( \rho \), the viscosity \( \mu \) and the thermal conductivity \( k \) are linear functions of the temperature:

\[
\rho(T) = \rho_m (1 - \beta (T - T_m(x))), \\
\mu(T) = \mu_m - \mu' (T - T_m(x)), \\
k(T) = k_m + k' (T - T_m(x)),
\]

where the subscript \( m \) means that the properties are evaluated at the mean temperature.

B. Conservation Equations

Considering a Boussinesq approximation (Incropera and DeWitt, 1999), the momentum equation can be written as

\[
0 = \left( -\frac{\partial p^*}{\partial x} \right) - \rho_m \beta (T - T_m) g + \frac{\partial}{\partial y} \left( \mu(T) \frac{\partial u}{\partial y} \right),
\]

where \( u \) is the velocity and \( p^* \) is define as