GROUP SOLUTION FOR UNSTEADY BOUNDARY LAYER FLOW OF A MICROPOLAR FLUID NEAR THE REAR STAGNATION POINT OF A PLANE SURFACE IN A POROUS MEDIUM

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Abstract — The transformation group theoretic approach is applied to the system of equations governing the unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface in a porous media. The application of a two-parameter group reduces the number of independent variables by two, and consequently the system of governing partial differential equations with boundary conditions reduces to a system of ordinary differential equations with appropriate boundary conditions. The possible form of potential velocity $U_e$ is derived in steady and unsteady cases. The family of ordinary differential equations has been solved numerically using a fourth-order Runge-Kutta algorithm with the shooting technique. The effect of varying parameters governing the problem is studied.

Keywords — Micropolar fluid, stagnation point, porous medium.

I. INTRODUCTION

Boundary layers of non-Newtonian fluids have received considerable attention in the last few decades. Boundary layer theory has been applied successfully to various non-Newtonian fluids models. One of these models is the theory of micropolar fluids introduced by Eringen (1964), Eringen (1966), Eringen (1977). In this theory, the micropolar fluid exhibits the microrotational effects and micro-inertia. The difficulty of the study of such fluid problem is the paucity of boundary conditions and the existence of deformable microelements as well as the time as the third independent variable. Many attempts were made to find analytical and numerical solutions by applying certain special conditions and using different mathematical approaches. Lok et al. (2003a) used the Keller-box method in conjunction with the Newton's linearization technique to study the unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface. Also, Lok et al. (2003b) studied the unsteady boundary layer flow of a micropolar fluid near the forward stagnation point of a plane surface by using the Newton's linearization technique of Keller-box method. Seshadri et al. (2002) used the implicit finite difference scheme to study the unsteady mixed convection flow in the stagnation region of a heated vertical plate due to impulsive motion.

On the other hand, studies with group method were used by Helal and Abd-el-Malek (2005), Abd-el-Malek et al. (2004). The mathematical technique which used in the present analyses is the two-parameter group transformation leads to a similarity representation of the problem. A systematic formulism is presented for reducing the number independent variables in systems which consist, in general, of a set partial differential equations and auxiliary conditions (such as boundary and/or initial conditions).

In the present work we consider the unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface in a porous media. The governing boundary layer equations have been transformed to ordinary differential equations via group analysis and these have been solved numerically using shooting technique. The effects of varying parameters governing the problem were studied.

II. MATHEMATICAL ANALYSIS

Let us consider the development of the two-dimensional boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface in a porous medium. The fluid which occupies a semi-infinite domain bounded by an infinite plane and remains at rest for time $t<0$ and starts to move impulsively away from the wall at $t=0$. In our analysis, rectangular Cartesian coordinates $(x,y)$ are used in which $x$ and $y$ are taken as the coordinates along the wall and normal to it, respectively. The flow configuration is shown schematically in Fig. 1.

![Fig.1. Physical model of the problem.](image-url)