SAMPLED-DATA MINIMUM VARIANCE FILTERING

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Abstract—This paper deals with the optimal solution to the sampled-data minimum variance filtering problem for linear systems with noise in the states and in the measurements. The solution is derived in the time-domain by using a fast sampling zero-order hold input discretization of the continuous time systems together with a lifting technique. The original sampled-data system is transformed into an equivalent LTI discrete-time system with infinite-dimensional input-output space. However, the designed filter is finite-dimensional. We derive both the existence conditions and the explicit expression of the desired filter and provide an illustrative numerical example.

Keywords—Filtering, lifting, T-periodic systems, sampled-data.

I. INTRODUCTION

Minimum variance filtering problems have been extensively studied via both the state space (Kalman 1960; Anderson and Moore, 1975; Shaked 1976) and the polynomial system approach (Wiener 1950; Roberts and Newmann, 1987; Ahlén and Sternad, 1991; and Grimble 1995). The design techniques are based on continuous-time (ct) or discrete-time (dt) system descriptions. However, in most applications a more realistic situation is that in which a digital filter must be designed to interact with ct systems. In such cases, the estimated signal is formed by the output of the dt filter through a zero-order hold device. The goal is to match the piecewise estimations to the desired ct signal. In general, there are two classic approaches used to design the corresponding ct systems. In the first approach consists of discretizing the ct system and designing a dt filter. In the second one, a digital implementation of the optimal filter obtained by a ct design is performed. Due to the intersample behaviour of the ct systems in both approaches, there is a serious performance degradation when the sampling is not fast enough. There is a third approach, called sampled-data design, in which the dt filter is designed taking into account the dynamics of the ct systems involved. The recent trends, such as techniques based on linear systems with jumps (Sun et al. 1993), and the lifting technique (Bamieh et al., 1991 and Hara et al., 1997), have been used to direct sampled-data design. Although these designs have been extensively used in feedback control systems, filters design received too little attention in spite of its importance in signal processing applications. Filtering sampled-data design has been investigated in the context of $H_\infty$ in Sun et al. (1993) and Kabamba et al. (1993), and in the $H_2$ context in Milocco and De Doná (1996), Wang et al. (2001), and Milocco and Muravchik (2003). In Wang et al. (2001), a filtering sampled-data design based on the Error Covariance Assignment criterion is proposed, while in Milocco and De Doná (1996) and Milocco and Muravchik (2003), a frequency domain approach to MIMO linear filter design for sampled-data system is presented by using a polynomial approach. In this paper, we extend the results obtained in Milocco and De Doná (1996) to design MIMO sampled-data filters in the time-domain. The proposed solution allows us to obtain the sampled-data minimum variance estimation of the states as well as optimal solutions for minimum variance sampled-data filtering problems such as deconvolution, prediction and smoothing.

The paper is organized as follows: In section II, we provide a suitable description of the multivariable sampled-data system, i.e. the ct subsystem followed by a sampling stage at intervals of $T$ sec., the dt subsystem or filter, and a holding device. Such systems can be represented with the help of $T$-periodic linear time-invariant systems. By means of a fast sampling zero-order hold input discretization of the ct systems together with a lifting technique, the original sampled-data system is transformed into an equivalent LTI dt system with infinite-dimensional input-output space. The cost to be minimized is defined as the averaged energy of the weighted output-error vector. In section III, the matrices of the dt filter state space representation are obtained such that the cost is minimized. In section IV, an example to illustrate the procedure is provided and finally, in section V, we present the conclusions.

II. PROBLEM FORMULATION

Consider the following ct time-invariant generalized system: