Robust Stability Test of Polytopic Family of Polynomials: 
The Dixon’s Resultant Method

Z.H. Wang‡, H.Y. Hu‡ and T. Küpper‡

†Institute of Vibration Engineering Research, Nanjing University of 
Aeronautics and Astronautics, 210016 Nanjing, P.R.China
‡Institute of Mathematics, University of Cologne, Wegertal 86-90, 50931 Cologne, Germany

Abstract—This paper deals with the D-stability test of a polytope of polynomials when the boundary \( \partial D \) of a given simple connected domain \( D \) in the complex plane is described by a polynomial equation, a problem that covers two special but important cases: Hurwitz stability and Schur stability of a polytope of polynomials. Based on the “Edge Theorem” and the method of Dixon’s resultant elimination, a new test approach is presented. By using the presented method, the stability test can be carried out by computing Dixon’s resultants and solving linear matrix equations. Two examples are given to demonstrate the approach.

Keywords—polytope, polynomials, robust stability, Dixon’s resultant.

I. INTRODUCTION

The robust stability of dynamic systems has drawn great attention over the past decades due to various uncertainties and errors always exist in the system modeling and parameter estimation. As is well-known, the stability analysis can be carried out by studying the root locations of the characteristic polynomials. Two important results for robust stability are due to Kharitonov (1979) who established a theory for the stability of interval families of polynomials, and to Bartlett et al. (1988) who developed the “Edge Theorem” for polytopic family of polynomials. Afterwards, different approaches, mainly based on the “Edge Theorem” or the “Zero-Exclusion Principle”, for checking the robust stability of a given polytope of polynomials were developed. A comprehensive description of robust stability analysis under parametric uncertainty was given in (Bhattacharyya et al., 1995).

The aim of this study is to present a new approach to test the D-stability of a given polytopic family \( \Omega \) when the boundary \( \partial D \) of \( D \) is described by a polynomial equation. Here, a family \( \Omega \) of polynomials is called D-stable if every polynomial in \( \Omega \) is D-stable, namely, all the zeros of each member of \( \Omega \) stay in \( D \). The classical Hurwitz stability and Schur stability fall into this category. Result that is most closely related to this paper is (Zeheb, 1989). Zeheb’s method is simple, but it cannot be applied to our case in general. Based on the “Edge Theorem”, the idea of the Dixon’s resultant for multivariate polynomials as in (Yang et al., 1996) is applied in this paper to carry out the D-stability test. With this method, we need only to compute several Dixon’s resultants and to solve some linear matrix equations. So the testing procedure is very simple.

II. PROBLEM FORMULATION

The polynomials under study are in the form

\[
p(\lambda) = a_0 \lambda^n + a_1 \lambda^{n-1} + \cdots + a_{n-1} \lambda + a_n
\]

where the real coefficients \( a_i, \ i = 0, 1, \cdots, n, \) depend linearly on some parameters that vary in given intervals respectively. Then, the family of polynomials is a polytope, generated by a finite number of polynomials \( p_1(\lambda), p_2(\lambda), \cdots, p_r(\lambda) \), as following

\[
\Omega = \text{conv}\{p_1(\lambda), p_2(\lambda), \cdots, p_r(\lambda)\}
\]

where \( p_i(\lambda), (i = 1, 2, \cdots, r) \) are in the form of Eq.(1) and are called vertex polynomials. Let \( D \) be a simple connected domain in the complex plane with the boundary \( \partial D \). The “Edge Theorem” states that the family \( \Omega \) of polynomials is D-stable if and only if all the “exposed” one-dimensional edge polynomials are D-stable. Here, each edge polynomial generated by two vertex polynomials \( p_i(\lambda) \) and \( p_j(\lambda) \) is a sub-family \( p_{ij}(\lambda, \mu) \) with a parameter \( \mu \)

\[
p_{ij}(\lambda, \mu) = (1 - \mu)p_i(\lambda) + \mu p_j(\lambda), \quad \mu \in [0, 1]
\]

In this paper, we assume that \( \partial D \) is described by a polynomial equation \( b(x, y) = 0 \). This covers two special but important cases. For the Schur stability of discrete-time systems, \( D \) is the open unit disk in the complex plane and \( b(x, y) = x^2 + y^2 - 1 \). For the Hurwitz stability of continuous-time systems, \( D \) is the open left half complex plane and \( b(x, y) = x \). Since \( p_{ij}(\lambda, \mu) \) is analytic with respect to \( \lambda \) and \( \mu \), a root \( \lambda = \lambda(\mu) \) of \( p_{ij}(\lambda, \mu) \) is continuous and cannot suddenly appear or disappear, or change its multiplicity at a finite point in the complex plane. With a variation of \( \mu \), thus, the sum of multiplicity of all roots of \( p_{ij}(\lambda, \mu) = 0 \) in \( D^c \), the complement of set \( D \), can