Abstract—This paper presents a new and less conservative condition for the robust stability test of discrete-time polytopic systems by using a descriptor system approach. The stability condition is formulated in terms of a set of linear matrix inequalities and can be easily adapted for robust controller synthesis. The developed results can be viewed as a discrete-time counterpart of the continuous-time one proposed by Cao and Lin (2004). This also enables us to offer a unified framework, namely the so-called descriptor system approach, for the analysis and synthesis of both discrete-time and continuous-time uncertain linear systems. Simulation examples are given to illustrate the theoretical results we established.

Keywords—Discrete-time systems, robust stability, parameter-dependent Lyapunov functions, descriptor system approach

I. INTRODUCTION

The problem of robust stability analysis and controller synthesis for uncertain systems has been extensively investigated in recent years. The Lyapunov-based approach is one of the most popular methods for solving this problem in the existing literature (see, e.g., Boyd et al., 1994; Feron et al., 1996; Gahinet et al., 1996; Oliveira et al., 1998; Geromel and Korogui, 2006; Su and Zhang, 2009; Zhang et al., 2010a; Zhang et al., 2010b; and the references therein). However, it is known that the traditional quadratic stability analysis usually leads to conservative results, especially in the case where the uncertainty is time invariant or slowly time-varying. To overcome this drawback, parameter dependent Lyapunov functions (PDLFs) were proposed in Feron et al. (1996) and Gahinet et al. (1996), where the stability conditions were formulated in terms of linear matrix inequalities (LMIs). Generally, the stability conditions based on PDLFs are less conservative than those resulted from a parameter independent Lyapunov functions (see, e.g., Daafouz and Bernussou, 2001; Lin et al., 2006; Gao et al., 2007).

The PDLFs-based approach has become a powerful tool in the analysis and design of linear uncertain systems since the pioneer work of Oliveira et al. (1999a, 1999b). By introducing a slack variable, Oliveira et al. (1999a, 1999b) proposed a new LMI condition for robust stability test of discrete polytopic systems. More importantly, the condition can be easily adapted for controller synthesis since it exhibits a kind of decoupling between the Lyapunov and the system matrices. The results were extended to the continuous-time case by Apkarian et al. (2001). Recently, another extension was proposed in Cao and Lin (2004) by applying a descriptor system approach, which was originally proposed by Fridman and Shaked (2002) to study the stability and $H_\infty$ control of time-delay systems. In general, there are two advantages of this approach. First, it can significantly reduce the conservatism by introducing some slack variables. Second, it can be easily applied to solve the problem of controller synthesis.

In this paper we revisit the problem of robust stability analysis and synthesis for uncertain discrete-time systems. We obtain a new and less conservative robust stability condition, which encompasses the known result proposed by Oliveira et al. (1999a) as a special case. The condition can also be viewed as a discrete-time counterpart of the continuous-time results given by Cao and Lin (2004). Also, the stability conditions can be easily adapted for controller synthesis of polytopic systems. Therefore, it is interesting to note that our results together with the work Cao and Lin (2004) present a new framework for the analysis and synthesis of uncertain linear systems. More precisely, we show that the descriptor system transformation is an efficient approach for the stability analysis and controller synthesis of both discrete-time and continuous-time polytopic systems. We finally use two numerical examples to illustrate the theoretical results.