BEHAVIOR OF THE SOLUTION OF THE TWO-PHASE STEFAN PROBLEM WITH REGARD TO THE CHANGING OF THERMAL COEFFICIENTS OF THE SUBSTANCE

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Abstract — We consider one-dimensional two-phase Stefan problems for a finite substance with different boundary conditions at the fixed faces. The goal of this paper is to determine the behavior of the free boundary and the temperature when the thermal coefficients of the material change.

We obtain properties of monotony with respect to the latent heat, the common mass density, the specific heat of each phase and the thermal conductivity of the liquid phase.

We show that the solution is not monotone with respect to the thermal conductivity of solid phase, in some cases, by computing a numerical solution through a finite difference scheme.

The results obtained are important in technological applications as the climate of buildings, the storage of energy in satellites and clothes and the transport of biological substances and telecommunications.

Keywords — Phase change material, Two phase Stefan problem, Finite difference method

I. INTRODUCTION

Several technological applications can be modeled through heat transfer problems with phase-change. The Phase Change Materials (PCMs) are substances whose phase-change temperature make them available to moderate oscillations of temperature and to store energy of another substance in contact with them (Jiji and Gaye, 2006). For this reason PCMs are used for the climate of buildings, the storage of energy in satellites and clothes and the transport of biological substances (Asako et al., 2002; Lamberg and Sim, 2003), among a variety of applications.

Usually, the technological way to select a PCM is through its phase-change temperature, but when there are several products in the correct range of temperature, it is necessary to study another properties of the substance, for example the thermal coefficients, in order to choose the most convenient PCM.

When we consider a packaging of a PCM that recovers an organic substance to be transported, it is essential to find the thickness of this pack to insure the optimal temperature of conservation in the organic substance during total time of transport (Bouciguez et al., 2001; Farid et al., 2004; Medina et al., 2004; Zalba et al., 2003; Živkovic and Fujii, 2001). Because the sizes of the pack (wide, length and height) are sufficiently greater than its thickness, we can assume that the heat transfer occurs in only one direction. Moreover, if we consider a material with one portion at solid state and the other at the liquid state, and take into account the environment conditions, we have an one dimensional two-phase Stefan problem (Lamberg, 2004; Lamberg et al., 2004).

In Olguín et al. (2007), we considered a one-dimensional one-phase Stefan problem for fusion of a semi-infinite material. We showed that, both, the temperature and the free boundary present a monotonous behavior with respect to the latent heat, the mass density and the specific heat. We also showed that the solution is not monotone with respect to the thermal conductivity.

At the present work we continue this line of research. We consider several two-phase one-dimensional Stefan problems for a finite material, with different boundary conditions at the two fixed faces, in order to determine the behavior of the solutions when the thermal coefficients change.

In Section 2 we study a phase-change problem with temperature specification on both fixed faces of the finite material and we obtain results of monotony for latent heat, mass density, specific heat of the solid and the liquid phase, and thermal conductivity of the liquid phase, by using the maximum principle (Protter and Weinberger, 1967).