NEHARI PROJECTION AND SOS IMPLEMENTATION IN REAL–TIME STABLE IDENTIFICATION

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Abstract— An adaptive identification algorithm based on Second Order Section (SOS) model structures is presented. The procedure guarantees stable transfer functions whenever the actual physical plant is stable, due to an optimal Nehari approximation step performed analytically online. The procedure is suitable for real time applications. Some synthetic and experimental examples illustrate the proposed algorithm.

Keywords— Nehari Projection, SOS Implementation, Real Time Identification

I. INTRODUCTION

Adaptive identification algorithms have been used in the area of adaptive control systems for a very long time, both for feedback (FB) and/or feedforward (FF) approaches (Goodwin and Sin, 1984; Tao, 2003). Usually for simplicity and computational speed in real time applications, parametric linear schemes have been implemented: RLS, NLMS, FXLMS, FULMS, as in the case of Active noise control (Kuo and Morgan, 1995), for example. Nevertheless the traditional assumptions in adaptive control: lack of perturbations or high frequency uncertain dynamics and minimum phase models, have generated at the end of the 80’s an intense work in the area of robustness of adaptive laws (Tao, 2003; Narendra, 1986; Ioannou and Sun, 1996). These have been extensively studied since then, and an excellent survey in this area can be found in Ortsga and Tang (1989).

Still then in adaptive identification, the stability of the resulting IIR model is generally not guaranteed, causing serious practical problems particularly in FF implementations. There are methods to convert IIR to FIR like the Nehari shuffle (Kootsookos et al., 1992) and a recent LMI optimal version in Yamamoto et al. (2002), but the error is usually greater and requires a larger number of parameters in general. The use of IIR filters instead has the potential to decrease the identification error due to the fact that it includes the pole dynamics. In addition, this class of filters are in certain applications more efficient in modelling signals and require smaller model orders (Rao, 1993). Therefore an IIR filter that can guarantee a stable behavior and can be used in real time applications is a necessary tool in practical situations.

On the other hand, numerical problems also arise in real time applications, depending on the structural representation of the model. Take for example an 11th. order stable filter implemented with three different model structures: zero–pole (ZP), state space (SS) and transfer function (TF), the latter in terms of numerator and denominator coefficients, as follows:

(ZP) \[ \prod_{i=1}^{m} \left( z_i z^{-1} - 1 \right) \]

(TF) \[ \sum_{i=0}^{m} z^{-i} b_i \]

(SS) \[ x_{k+1} = A x_k + B y_k \]
\[ y_k = C x_k + D y_k \]

The complexity of each model is \( O(m^2) \) in the case of SS and \( O(m) \) in the other two cases, therefore from this point of view, the ZP and TF structures are more efficient. Nevertheless, it is a well known fact that the pole locations in the case of the TF structure, particularly in high order models, are significantly modified, even producing unstable poles \(|p_i| > 1\), as illustrated in Table 1. On the other hand, it is easier to use the TF representation as the difference equation which implements the filter in real time, as follows:

\[
y_k = \frac{1}{\alpha_0} \left[ b_0 u_k + \cdots + b_m u_{k-m} \right] \\
- \alpha_1 y_{k-1} - \cdots - \alpha_m y_{k-m} \]

(1)

Therefore, the TF representation has advantages in terms of complexity and implementation, but serious disadvantages in terms of perturbations of pole locations, at least in high order models.

The solution to this problem is obtained by a series connection of Second Order Sections (SOS), which is an adequate way of implementing filters in real time. The SOS structure is numerically more efficient than the plain TF structure due to the fact that it has a 2nd. order numerator and denominator, therefore preserving the original pole–zero locations. In addition, cascade-forms of SOS provide an attractive realization