THE CLOSING FUNCTION IN THE WATERHAMMER MODELING

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Abstract—The closing valve law is a mathematical function that describes the speed variation of the fluid as it is closing. This reduction of the speed determines the shape of the pressure wave during the development of the water-hammer. A wide variety of closing modes exists, depending on the valve type and their operation, each one is mathematically given by a function. A generic function was formulated that allows to model an extensive variety of closing laws by means of a polygonal segmented structure. An algorithm has been generated that includes this closing law as boundary condition for describing the transitory. The pressure wave shape and amplitude depends on closing function in a unique relationship.

Keywords—Water-hammer, valve, closing law, pressure wave, transitory.

I. INTRODUCTION
The water-hammer has its origins in accidental or programmed perturbations on hydraulic systems. The operation of a valve generates a transitory in the hydrodynamic system that produces changes on the flow conditions. These changes are observed in the oscillatory behavior that shows the pressure and the fluid speed, an alternate succession of crests and troughs that attenuates in time. This transitory is known as water-hammer.

The earlier studies on the water-hammer phenomena have been found in the works of Young (1808), Wertheim (1848) and Michaud (1878) among others. Joukowski (1900, 1904) published the results of the experimental studies carried out on the water system distribution of Moscow. In that work it was extended the transitory description to the total time of duration. Established a rational expression for the complex variations of pressure that experiences a pipe net taking into account the reflection and transmission in pipe boundaries, and introduced the pipe period concept. These works were based on the observations and developments of Wertheim (1848), von Helmholtz (1847), Korteweg (1878) and Lamb (1898).

During that period (1903-1913) Lorenzo Allievi was an active investigator in this theme, arriving to the same results as Joukowski, assuming pipe flow without friction, uniform section, homogeneous wall and uniform distribution of speed of the fluid. Allievi has developed the wave equations, rejecting the convective terms and solving it by the general method proposed by Riemann and D’Alembert. Allievi (1925) has extended the Joukowski results to the cases of non instantaneous valve closing, that is to say, closing times higher than the pipe period being able to predict the variations of pressure along the pipe and not only on the valve (Murga and Molina; 1997).

Wood (1938) introduced the Heaviside operational calculus and presented a solution for a simple pipe with instantaneous valve closing. Rich (1945) proposed the use of the Laplace-Mellin transform for the same system.

With the advancement in computing technology appeared the firsts numeric methods for the Water-hammer modeling (Harding, 1966). The Method of the Characteristics is a particularly appropriate technique for the solution of hyperbolic partial differential equations (Abbott, 1966). Gray (1953, 1954), Ezekial and Paynter (1957) and Streeter et al. (1962, 1967, 1972, 1983) have found useful the use of this technique.

In Europe, Fox (1968), Evangelisti (1969) and Swaffield (1970) were the precursors in the use of this method, which has settled down as a standard technique for the transitory analysis (Brunone et al., 1991).

Some authors have indicated the influence of the closing perturbation on the pressure wave transient, however the closing functions that have been included in successive works in models of the Water-hammer are restricted to the instantaneous, the lineal and the cosecoidal closing (Hager, 2001).

The objective of the present work is the development of a mathematical model that takes into account the closing law, and the formulation of an algorithm that allows to describe a wide range of closing functions.

II. THE MODEL
The most common cause of Water-hammer is an accidental (a pump getting out of service) or programmed (valve closing) flow perturbation. The location where the perturbation takes place is considered a frontier of the system. Before and after this location, the flow changes are different. A π angle shift appears between the upstream and downstream pressure waves that travel across the pipe. These characteristics allow to evaluate the valve closing (or any device that generates a perturbation) as a boundary condition.

A water-hammer mathematical model was developed using a simple system, constituted by a reservoir, a single horizontal constant diameter conduction with a valve at the pipe end (Fig. 1).

The set of equations that define the analytic pattern of water-hammer is: