INFLUENCE OF MAGNETIC FIELD ON NATURAL CONVECTION FLOW NEAR A WAVY CONE IN POROUS MEDIA

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Abstract — The results of a study of effect of magnetic field on natural convection along an isothermal wavy cone embedded in a fluid-saturated porous medium are obtained. A coordinate transformation is used to transform the complex wavy conical surface to a smooth conical surface, and the transformed non-similar boundary layer governing equations are then solved numerically by means of the Runge-Kutta integration scheme with the Newton-Raphson shooting method. The boundary layer under consideration is concerned with the regime where the Darcy-Rayleigh number Ra is very large. Detailed results of the effect of magnetic field, half cone angle, and the sinusoidal wavy surface on the velocity, temperature and the wall heat flux are presented.

Keywords — Magnetic field, wavy cone, porous media.

I. INTRODUCTION

The study and analysis of heat transfer in porous media have been the subject of many investigations due to their frequent occurrence in industrial and technological applications. Examples of some applications include geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, and many others. There has been considerable work done on free convection flow over conical surfaces which is based on the laminar boundary-layer approach. See, for instance, Cheng and Minkowycz (1977), Chamkha (1996), Kafousias (1992), Yih (1999), Alamgir (1979), Na and Chiou (1979a, b), El-kabeir et al. (2006), Hery and Grosh (1965) and Roy (1974), and many other papers can be found in Nield and Bejan (1999). Most early studies on convection heat transfer in porous media have used regular surfaces.

The study of heat transfer near irregular surfaces is of fundamental importance; that is because it is often met in many practical applications and devices such as flat-plate solar collectors and flat-plate condensers in refrigerators. The presence of roughness elements disturbs the flow past surfaces and alters the heat transfer rate. Yao (1983) studied the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surfaces, in Newtonian fluids. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces. Rees and Pop (1994a, b) and (1995) carried out some studies to analyze natural convection from vertical and horizontal wavy surfaces embedded in a porous medium. Hady et al. (2006) analyzed the problem of MHD free convection flow along a vertical wavy surface in presence of magnetic field and generation absorption. Pop and Na (1994) and (1995) studied the natural convection flow along a vertical wavy cone and a frustum of a wavy cone in porous media. Hossain and Pop (1996) studied the magnetohydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface in Newtonian fluids. Cheng (2000a, b) and Cheng (2007) reported the phenomenon of natural convection heat and mass transfer near a vertical wavy surface and near a wavy cone and a frustum wavy cone with constant wall temperature and concentration in a porous medium.

Motivated by the works mentioned above, the steady, laminar, free convection flow along a wavy cone and immersed in an electrically conducting fluid-saturated porous medium in the presence of a transverse magnetic field is considered. The surface temperature of the cone is assumed to be constant. The applied magnetic field is assumed to be uniform and the magnetic Reynold's number is assumed to be small so that the induced magnetic field can be neglected. In addition, it is assumed that the external electric field is zero and the electric field due to polarization of charges is negligible.

II. MATHEMATICAL ANALYSIS

Consider the steady-state boundary layer flow near a wavy cone with transverse sinusoidal undulations embedded in a fluid-saturated porous medium, as illustrated in Fig. (1). The wavy surface profile is given by

\[ \hat{y} = \hat{\delta} = \hat{a} \sin (\pi \hat{\delta} / \ell), \]

where \( \hat{a} \) is the amplitude of the wavy surface, \( \ell \) is the length scale of the wavy surface. The origin of the coordinate system is placed at the leading edge of the wavy cone. We assume that the temperature of the wavy cone is held at constant value \( T_w \) and a uniform ambient temperature \( T_\infty \). The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

Based on Boussinesq approximations, the equations governing the steady-state conservation of mass, momentum, and energy for Darcy flow through a homogeneous porous medium near the wavy cone can be given as