AN OPTIMAL APPROACH TO THE MULTIPLE-DEPOT HETEROGENEOUS VEHICLE ROUTING PROBLEM WITH TIME WINDOW AND CAPACITY CONSTRAINTS

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Abstract. The vehicle routing problem (VRP) has become a crucial industrial issue for its impact on product distribution costs. Though quite important in practice, the time-constrained version of the VRP accounting for several types of vehicles and m-depots, called the extended VRP with time windows (m-VRPTW), has received less attention. Since it is an NP-hard problem, most of the current approaches to m-VRPTW are heuristic, thus providing good but not necessarily optimal solutions. This work presents a novel MILP mathematical framework for the m-depot heterogeneous-fleet VRPTW problem. The new optimization approach permits to find both the optimal vehicle route/schedule and the fleet size by choosing the best set of preceding nodes for each pick-up point. To get a significant reduction on the problem size to tackle larger m-VRPTW problems, some elimination rules have been embedded in the MILP framework. When applied to a pair of examples, it was observed a remarkable saving in computer costs with regards to prior VRPTW optimization methods.

Keywords: VRPTW, MILP formulation, soft/hard time windows

I. INTRODUCTION

In recent years, vehicle routing problems (VRP) have received much attention due to the importance of selecting efficient distribution strategies. Surveys on VRP problems can be found in Bodin et al. (1983) and Desrosiers et al. (1995). Vehicle routing problems with time windows (VRPTW) have recently become an area of intensive research since time windows naturally arise from realistic considerations and their impact on the optimal solution must be taken into account. Similarly to VRP, VRPTW problems are NP-hard too. Exact algorithms developed for solving VRPTW are described in Kolen et al. (1987), Desrochers et al. (1992) and Fisher et al. (1997). Though heuristic techniques still remain as the unique tool available for solving very large-scale VRP, optimization methods are becoming more effective to deal with problems of moderate size. Nevertheless, traditional optimizing formulations still require an exceptionally large number of variables and constraints to represent real routing problems. In order to widen the scope for optimizing approaches, this work introduces a new MILP formulation for the m-depot VRPTW problem, also called m-VRPTW, based on a continuous time domain representation and a separate handling of vehicle assignment and node sequencing decisions.

II. PROBLEM FORMULATION

Let the road network be described by \( I, P, A \) with \( I = \{i_1, i_2, ..., i_k\} \) denoting the set of nodes or customers, and \( P = \{p_1, p_2, ..., p_l\} \) representing the set of depots where the vehicles \( V = \{v_1, v_2, ..., v_m\} \) are housed. Nodes and depots are connected through a network of minimum cost arcs \( A = \{d_{ij} / i, j \in (I \cup P)\} \). Each node represents a client that has a non-negative commodity production \( p_j \) to be picked up and transported to a depot \( p \in P \). There is a matrix of unit routing costs \( C = \{c_{ij}\} \) and a matrix of vehicle speeds \( \Gamma = \{\gamma_v\} \) associated to the set \( A \). Each node has its time window \([a_i, b_i]\), where \( a_i \) is the earliest arrival time and \( b_i \) is the latest visiting time. The service time on node \( i \) by vehicle \( v \) is \((t_{fi} + p_{ri}) / r_v\), where \( t_{fi} \) is the fixed stop time on node \( i \) and \( r_v \) is the loading rate of goods on vehicle \( v \). The solution to the problem must comply the following constraints: (i) each route starts and ends on the same depot; (ii) each node belongs to exactly one route; (iii) the total amount of commodity assigned to vehicle \( v \) must never exceed its capacity \( q_v \); (iv) the duration of the trip for any vehicle \( v \) should be shorter than the maximum allowed routing time \( t_v^{max} \); (v) the node \( i \) should be serviced within the time window \([a_i, b_i]\). The problem goal is to minimize the total cost of providing pickup service to every node. Three types of costs are considered in the objective function: fixed costs for using vehicles, traveling costs along the selected routes and penalty costs fining soft time-window and maximum routing time constraints violations.

III. THE MATHEMATICAL MODEL

Assignment of nodes to vehicles

Each node must belong to exactly one tour.

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\sum_{v \in V} Y_{iv} = 1 \quad \forall i \in I
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