NUMERICAL EXPERIMENTS ON PARAMETER ESTIMATION IN ACOUSTIC MEDIA USING THE ADJOINT METHOD

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Abstract—We present an algorithm to solve the problem of estimation of the wave speed in a layered acoustic medium. It is based on the adjoint method. Even though it is a fast algorithm, it is able to detect approximately the interfaces and estimate the amplitudes of the speed parameter. Numerical examples illustrate the application of the algorithm in seismic inversion.

Keywords—inverse problems, seismic inversion, acoustic equation, finite element methods.

I. INTRODUCTION

The purpose of this work is to present a fast algorithm to estimate the wave speed in a layered acoustic bounded domain.

In previous papers we treated this estimation problem using a Newton-type method on a quasilinearization algorithm. Newton-type methods are time consuming. In order to improve the performance we present a steepest descent procedure. We use the gradient of the linearized cost functional which is computed from its continuous version. The gradient is calculated using the adjoint of the Gâteaux derivative of the solution of the equation with respect to the parameter.

Most of the necessary mathematics is presented in a previous paper (Fernández-Berdaguer, 1998). There the proofs are done for absorbing boundary conditions at the top and bottom of the domain. Here we use more realistic ones: a Dirichlet boundary condition at the top of the boundary, i.e., the air-solid interface and an absorbing boundary condition at the bottom boundary.

The present work describes the implementation of the method for the one-dimensional case in \( \Omega = (0, 1) \). This simple case shows the most important aspects of the scheme, for example, the detection of the interfaces and the estimation of the parameter amplitudes, with minor computational burden.

This paper presents a fast estimation algorithm to solve a generalization of the velocity inversion procedure first described in Fernández-Berdaguer et al., 1996. In that paper we considered a layered medium consisting of layers of known thickness. Our method here allows us to attack problems where no a priori information is available about the parameterization of the unknown parameter function. The theory developed in Fernández-Berdaguer (1998) allows for the wave speed \( c(x) \) to be an arbitrary function in \( L^\infty(\Omega) \); thus at the discrete level, \( c(x) \) may take different values at each point of the spatial discretization.

It is worth noting that even when our work is intended for applications in geophysical exploration, the analysis and algorithms presented here are also of interest in other fields such as non-destructive material testing (Burk and Weiss, 1979), polymer physics (Ferry, 1970) and ocean acoustics (Stoll, 1977).

The organization of the paper is as follows: in section 2 we present the direct model, the inverse problem and the sensitivity equations. In section 3 we state the algorithm and its implementation. Finally, in section 4 we present some numerical examples.

II. THE DIRECT MODEL, THE ESTIMATION PROBLEM AND THE SENSITIVITY EQUATIONS

A. The direct model

Let \( \Omega = (0, 1), \Gamma = \partial \Omega = \{0, 1\} \). The problem is to estimate the parameter \( c(x) \) in the usual model for